CALCULATING THE NONSTEADY HEAT CONDUCTION OF AN UNBOUNDED PLATE, WITH MIXED BOUNDARY CONDITIONS

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The solution for the heat-conduction equation

$$\frac{\partial t(x,\ \tau)}{\partial \tau} = a\frac{\partial^2 t(x,\ \tau)}{\partial x^2} \quad \left(-\frac{l}{2} < x < \frac{l}{2},\ \tau > 0\right)$$

with the boundary condition

$$t(x,\ 0) = t_0 = \mathrm{const}$$

and the boundary conditions by means of which we take into consideration the simultaneous effect of radiation and convection heat flows

$$\frac{\partial t\left(-\frac{l}{2},\ \tau\right)}{\partial x} = -\frac{\alpha_1}{\lambda_1}\left[t_1 - t\left(-\frac{l}{2},\ \tau\right)\right] + \frac{q_1}{\lambda_1},$$

$$\frac{\partial t\left(\frac{l}{2},\ \tau\right)}{\partial x} = \frac{\alpha_2}{\lambda_2}\left[t_2 - t\left(\frac{l}{2},\ \tau\right)\right] + \frac{q_2}{\lambda_2},$$

achieved by an operator method, has the form

$$\frac{t(x,\ \tau) - t_0}{t_0} = \frac{a_1 + a_2 + 0.5(a_1\,\mathrm{Bi}_2 + a_2\,\mathrm{Bi}_1) + (a_2\,\mathrm{Bi}_1 - a_1\,\mathrm{Bi}_2)\frac{x}{l}}{\mathrm{Bi}_1 + \mathrm{Bi}_2 + \mathrm{Bi}_1\mathrm{Bi}_2}$$

$$+ 2\sum_{(\mu_n)}\left\{a_1\left[\cos\mu_n\left(\frac{1}{2} - \frac{x}{l}\right) + \frac{\mathrm{Bi}_2}{\mu_n}\sin\mu_n\left(\frac{1}{2} - \frac{x}{l}\right)\right] + a_2\left[\cos\mu_n\left(\frac{1}{2} + \frac{x}{l}\right) + \frac{\mathrm{Bi}_1}{\mu_n}\sin\mu_n\left(\frac{1}{2} + \frac{x}{l}\right)\right]\right\}$$

$$\times\left\{(\mathrm{Bi}_1\mathrm{Bi}_2 + \mathrm{Bi}_1 + \mathrm{Bi}_2 - \mu_n^2)\cos\mu_n - (\mathrm{Bi}_1 + \mathrm{Bi}_2 + 2)\,\mu_n\sin\mu_n\right\}^{-1}\exp(-\mu_n^2\,\mathrm{Fo})$$

when

$$a_1 = (\mathrm{Bi}_1 - \mathrm{Ki}_1)\left(\frac{t_1}{t_0} - 1\right), \quad a_2 = (\mathrm{Bi}_2 - \mathrm{Ki}_2)\left(\frac{t_2}{t_0} - 1\right)$$

and

$$a_1^2 - a_2^2 \neq 0; \quad \mathrm{Bi}_1^2 + \mathrm{Bi}_2^2 \neq 0.$$

The roots μ_n of the characteristic equation

$$\frac{\mathrm{tg}\,\mu}{\mu} = \frac{\mathrm{Bi}_1 + \mathrm{Bi}_2}{\mu^2 - \mathrm{Bi}_1\mathrm{Bi}_2}$$

are presented in Table 1.

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TABLE 1. The Roots μ_n of the Characteristic Equation

0.00	0.000 3.141 6.283		8.00	10.0	20.0	40.0		80.0	100.0	Bi_2 ← ↓ Bi_1
0,20	0,433 3,204 6,315	0,622 3,264 6,346	2,529 5,141 7,870	2,558 5,223 7,969	2,684 5,423 8,234	2,743 5,540 8,407	2,763 5,581 8,471	2,773 5,602 8,503	2,779 5,615 8,523	8,00
0,40	0,593 3,264 6,346	0,750 3,322 6,377	0,866 3,377 6,408	2,628 5,307 8,067	2,738 5,511 8,335	2,799 5,631 8,510	2,820 5,673 8,574	2,830 5,694 8,607	2,837 5,708 8,627	10,0
0,60	0,705 3,320 6,377	0,848 3,377 6,408	0,956 3,431 6,438	1,044 3,483 6,468	2,858 5,726 8,612	2,923 5,853 8,794	2,946 5,898 8,896	2,958 5,921 8,986	2,965 5,935 8,916	20,0
0,80	0,791 3,374 6,407	0,926 3,429 6,438	1,030 3,482 6,468	1,116 3,533 6,498	1,186 3,581 6,527	2,992 5,986 8,983	3,016 6,033 9,052	3,028 6,057 9,088	3,036 6,072 9,110	40,0
1,00	0,860 3,426 6,437	0,990 3,478 6,468	1,092 3,531 6,497	1,176 3,580 6,527	1,247 3,628 6,556	1,307 3,673 6,585	3,040 6,081 9,123	3,052 6,106 9,159	3,060 6,120 9,181	60,0
2,00	1,077 3,644 6,578	1,197 3,692 6,607	1,295 3,739 6,636	1,378 3,785 6,665	1,449 3,829 6,693	1,509 3,871 6,720	1,721 4,058 6,951	3,065 6,130 9,196	3,072 6,145 9,218	80,0
4,00	1,265 3,935 6,814	1,382 3,980 6,841	1,480 4,023 6,869	1,564 4,065 6,896	1,637 4,106 6,922	1,700 4,146 6,948	1,926 4,322 7,073	2,154 4,578 7,287	3,080 6,160 9,240	100,0
6,00	1,350 4,112 6,992	1,467 4,155 7,019	1,566 4,197 7,045	1,651 4,238 7,071	1,725 4,278 7,097	1,790 4,316 7,123	2,025 4,489 7,244	2,265 4,744 7,454	2,385 4,911 7,618	6,00
8,00	1,398 4,226 7,126	1,515 4,269 7,152	1,615 4,311 7,178	1,700 4,351 7,204	1,776 4,391 7,229	1,842 4,429 7,254	2,082 4,601 7,374	2,331 4,856 7,581	2,455 5,025 7,744	8,00
10,0	1,429 4,306 7,228	1,546 4,348 7,254	1,646 4,390 7,280	1,733 4,430 7,305	1,808 4,469 7,330	1,875 4,507 7,355	2,119 4,679 7,474	2,373 4,935 7,679	2,501 5,106 7,842	10,0
20,0	1,496 4,491 7,495	1,614 4,534 7,521	1,714 4,575 7,546	1,802 4,615 7,571	1,880 4,654 7,596	1,948 4,692 7,620	2,199 4,864 7,738	2,466 5,124 7,942	2,603 5,301 8,106	20,0
40,0	1,533 4,598 7,665	1,650 4,640 7,690	1,752 4,681 7,715	1,840 4,721 7,740	1,918 4,760 7,765	1,987 4,798 7,789	2,243 4,971 7,907	2,517 5,235 8,112	2,658 5,414 8,277	40,0
60,0	1,545 4,635 7,726	1,663 4,677 7,751	1,764 4,718 7,776	1,653 4,759 7,801	1,931 4,798 7,826	2,000 4,836 7,851	2,258 5,009 7,968	2,535 5,274 8,174	2,677 5,455 8,340	60,0
80,0	1,551 4,654 7,757	1,670 4,670 7,782	1,770 4,737 7,807	1,860 4,777 7,833	1,937 4,816 7,857	2,007 4,854 7,881	2,265 5,028 7,999	2,543 5,293 8,205	2,686 5,475 8,371	80,0
100,0	1,555 4,666 7,776	1,673 4,708 7,801	1,775 4,749 7,827	1,863 4,789 7,851	1,948 4,828 7,876	2,012 4,866 7,901	2,270 5,039 8,018	2,548 5,305 8,224	2,693 5,487 8,391	100,0
Bi_1 ↑ → Bi_2	0.000	0.20	0.40	0.60	0.80	1.00	2.00	4.00	6.00	Bi_1